Gauss' Law

Gauss' law (or Gauss theorem) states that the net outward electric flux through any closed surface in an electric field is equal to $\frac{1}{\epsilon_0}$ times the total charge or zero according to the closed surface encloses the charge or not, where ϵ_0 is the permitivity of the medium.

Thus for a closed surface S enclosing a charge q, the law can be mathematically expressed as

$$\oint_S \vec{E}\cdot\vec{ds} = \frac{1}{\epsilon_0}q, \quad \text{when S encloses q}$$

$$= 0 \quad \text{when S does not enclose q.}$$

Here $\vec{E}$ represents the electric field at the centre of an elementary area ds. The above equation often known as integral form of Gauss' law.

Gauss' Law in Differential form:

From the integral form

$$\oint_E \vec{E}\, d\vec{s} = \frac{q}{\epsilon_0} \quad \text{--- (1)}$$

According to Gauss' diverge theorem

$$\oint_S \vec{E}\cdot d\vec{s} = \int_V (\nabla\cdot\vec{E})\, dv \quad \text{--- (2)}$$

where 'v' represents the volume having surface S as the boundary. For continuous charge distribution

$$q = \int_V \rho\, dV \quad \text{--- (3)}$$

where ρ is the volume charge density

$$\therefore \int_V \nabla\cdot\vec{E}\, dv = \frac{1}{\epsilon_0}\int_V \rho\, dv$$

or $$\int_V \left(\nabla\cdot\vec{E} - \frac{\rho}{\epsilon_0}\right) dv = 0$$

or $$\nabla\cdot\vec{E} - \frac{\rho}{\epsilon_0} = 0 \quad \text{or} \quad \nabla\cdot\vec{E} = \frac{\rho}{\epsilon_0} \quad \text{--- (4)}$$

Equation (4) is known as the differential form of Gauss' law.

Gauss's law of magnetostatics:

The rate of change of magnetic flux through a closed surface is always zero.

$$\oint \vec{B}\cdot d\vec{s} = 0$$

This also signifies that monopole cannot exist.